

Sequential state discrimination with quantum correlation

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Abstract We study the sequential unambiguous state discrimination (SSD) with an ancilla system for initial states prepared with arbitrary *priori* probabilities. The results are compared with some strategies that allow classical communication. It's found that the optimal success probability of SSD is enhanced by the deviation from equal probability. The proportion of left (right) discord in their total is positively (negatively) correlated with the information extracted by Bob. Identifying unequal prior states calls for less quantum correlation than the equal prior case.

Keywords Sequential state discrimination · Entanglement · Discord

1 Introduction

It is crucial to make clear the roles of quantum correlations in quantum information processing. Such correlations have been widely investigated in various perspectives. Many of them are related to quantum entanglement [1], Bell non-locality [2], and quantum discord [3,4]. The results in studies [5,6] show that the algorithm for deterministic quantum computation with one qubit (DQC1) can surpass the performance of the corresponding classical algorithm in the absence of entanglement between the control qubit and a completely mixed state. Thus, the entanglement which had been regarded as the only resource for demonstrating the superiority of quantum information processing [1,7] is completely redundant [8]. The quantum discord, which gives a measurement

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of the nonclassical correlations and can exist in a separable state, is considered to be the key resource in this quantum algorithm and has gained wide attention in recent years [9, 10]. Another type of quantum correlations called dissonance and put forward by [9] measures the nonclassical correlations with entanglement being completely eliminated. For a separable state, its dissonance is exactly equal to the quantum discord which plays a key role in the computational process.

Unambiguous discrimination among linearly independent nonorthogonal quantum states is a fundamental subject in quantum information theory [11, 12, 13, 14, 15] which is the second example after DQC1 aided only by quantum dissonance rather than entanglement [16, 17]. The distinction of two unknown nonorthogonal states of a qubit, $|\Psi_1\rangle$ and $|\Psi_2\rangle$, prepared by Alice and sent to Bob, may sometimes fail as the price to pay for no error. Thus, the measurement has three possible outcomes, 1, 2, 0 corresponding to $|\Psi_1\rangle$, $|\Psi_2\rangle$ and failure outcome respectively. The topic of extracting information from a quantum system by multiple observers have been studied by [18, 19, 20].

Among them is the theory of nondestructive sequential state discrimination (SSD) put forward by Bergou *et al* [20]. Namely, after his discrimination, Bob send the qubit to a next observer Charlie, who also performs an unambiguous discrimination measurement to determine which state is prepared by Alice. The success probability of Charlie's measurement depends on the overlap between the two possible states that Bob's measurement leaves in the qubit. Pang *et.al.* [8] study the theory by using the language of system ancilla and explore the roles of quantum correlations in such procedure. It is indicated that the optimal success probability prepared for both Bob and Charlie to recognize between the two states relies on the quantum dissonance which equals to discord as the entanglement is absent. However, only the special case with equal prior is concerned in the two published papers. The aim of the present work is to investigate the influence of *a priori* probabilities and to check whether the existing conclusions hold for the unequal prior cases.

The detailed discussions of SSD is presented in Sec. 2. We compare the results of SSD with other three protocols that allows classical communication in Sec. 3. The effects of quantum correlations is discussed in Sec. 4. The final section is a summary.

2 Sequential state discrimination

We now consider SSD with arbitrary *priori* probabilities (the protocol is shown in Fig.1). A qubit A sent by Alice to Bob was prepared in one of nonorthogonal states $|\Psi_1\rangle$ or $|\Psi_2\rangle$ with their corresponding *priori* probabilities P_1, P_2 ($P_1 + P_2 = 1$) respectively. After a joint unitary transformation U_b between A and B is performed, the state of the composite system is obtained by Bob

$$\begin{aligned} P_1 : U_b |\Psi_1\rangle |0_b\rangle &= \sqrt{q_1^b} |\chi_1\rangle |0\rangle_b + \sqrt{1 - q_1^b} |\phi_1\rangle |1\rangle_b, \\ P_2 : U_b |\Psi_2\rangle |0_b\rangle &= \sqrt{q_2^b} |\chi_2\rangle |0\rangle_b + \sqrt{1 - q_2^b} |\phi_2\rangle |2\rangle_b, \end{aligned} \quad (1)$$

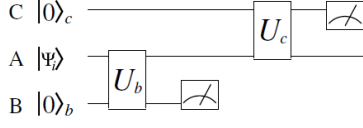


Fig. 1 Protocol for SSD. A qubit A is prepared by Alice in one of the two states $|\Psi_1\rangle, |\Psi_2\rangle$ with *priori* probabilities P_1, P_2 respectively. After the qubit is sent to Bob, a joint unitary operation is performed between the qubit A and an auxiliary qutrit B , followed by a von Neumann measurement on the qutrit. The state discrimination is successful if the outcome is 1 (for $|\Psi_1\rangle$) and 2 (for $|\Psi_2\rangle$), but unsuccessful if the outcome is 0. Then the qubit which is in the postmeasurement state is sent to Charlie. Charlie performs a similar joint unitary operation U_c between it and his qutrit C and makes an optimal unambiguous discrimination measurement on C .

with $|0\rangle_b, |1\rangle_b, |2\rangle_b$ as the basis of the ancilla and $|\chi_{1,2}\rangle$ and $|\phi_{1,2}\rangle$ as the pure states of A . If the state $|\Psi_i\rangle$ ($i = 1, 2$) is input, after his measurement, we say Bob succeeds in discrimination if the ancilla collapses to $|i\rangle_b$ and the qubit to $|\phi_i\rangle$; while he fails if the ancilla is onto $|0\rangle_b$ and A to $|\chi_i\rangle$. Because unitary operation doesn't change the inner product of the two states $\langle\Psi_1|\Psi_2\rangle = s$, so the failure state $|\chi_i\rangle$ satisfies the constraint $\sqrt{q_1^b q_2^b} \langle\chi_1|\chi_2\rangle = s$. Without considering the loss of generality, we take the overlap s to be real ($0 \leq s \leq 1$). If we set $\langle\chi_1|\chi_2\rangle = t$, the success probability of Bob is obtained as

$$P_b = P_1(1 - q_1^b) + P_2(1 - q_2^b). \quad (2)$$

Considering the constraint $q_1 q_2 = \frac{s^2}{t^2}$ and $q_1 \leq 1, q_2 \leq 1$, the following inequality should be satisfied $q_1 \geq \frac{s^2}{t^2}, q_2 \geq \frac{s^2}{t^2}$. Then the maximal success probability P_{bmax} of Bob has two candidates which can be achieved for different values of P_1 . The two cases are (i): $0 \leq P_1 < \frac{s^2}{s^2+t^2}$ and (ii): $\frac{s^2}{s^2+t^2} \leq P_1 \leq \frac{1}{2}$ with the maximal success probability

$$(i) : P_{bmax} = (1 - P_1)(1 - \frac{s^2}{t^2}); \quad (3a)$$

$$(ii) : P_{bmax} = 1 - 2\sqrt{P_1 P_2} \frac{s}{t}. \quad (3b)$$

P_{bmax} is attained for $q_1^b = 1, \sqrt{\frac{P_2}{P_1}} \frac{s}{t}$ corresponding to the two cases respectively. A special case ($P_1 = P_2 = 1/2, P_{bmax} = 1 - \frac{s}{t}$) which is consistent with the results in [8] corresponds to the minimum P_{bmax} of Bob. While the maximum measurement probability in case (i) causes a fact that one of the initial states ($|\Psi_1\rangle$ or $|\Psi_2\rangle$) corresponds to a result which is bound to fail is ignored. As P_1 approaches 0 or 1, the maximum success probability of the measurement occurs. Outwardly Alice has only one state ($|\Psi_1\rangle$ or $|\Psi_2\rangle$), but Bob doesn't know the truth because the classical communication is prohibited. If the outcome is 0, he is still unsuccessful. Thus the success probability can't reach 1 despite only one state left by Alice.

Then, the qubit A is sent to the second observer Charlie. We assume that Charlie knows Bob's protocol. Because only pure states can be discriminated by Charlie, which requires $|\chi_i\rangle = |\phi_i\rangle$, therefor the states in Eq.(1) become

$$\begin{aligned} P_1 : U_b|\Psi_1\rangle|0_b\rangle &= |\phi_1\rangle|\alpha_1\rangle_b, \\ P_2 : U_b|\Psi_2\rangle|0_b\rangle &= |\phi_2\rangle|\alpha_2\rangle_b, \end{aligned} \quad (4)$$

where $|\alpha_i\rangle_b = \sqrt{q_i^b}|0\rangle_b + \sqrt{1-q_i^b}|i\rangle_b$ with $i = 1, 2$. After the qubit is received, Charlie makes a similar joint unitary operation U_c between the qubit A from Bob and his auxiliary qutrit C as is shown by the Eq.(1) with the parameters q_1^c and q_2^c , followed by an optimal unambiguous measurement [20] on C . The optimal success probability can be derived as

$$(i) : P_{cmax} = (1 - P_1)(1 - t^2), \quad (5a)$$

$$(ii) : P_{cmax} = 1 - 2\sqrt{P_1 P_2}t, \quad (5b)$$

corresponding to the two cases (i): $0 \leq P_1 < \frac{t^2}{1+t^2}$ and (ii): $\frac{t^2}{1+t^2} \leq P_1 \leq \frac{1}{2}$ respectively.

For fixed s , it's found that P_{cmax} is negatively related to P_{bmax} . The overlap t ($s \leq t \leq 1$) is positively correlated with the information extracted by Bob after his measurement. Thus when t increases, the information left for Charlie to identify the states decreases on the contrary, that's the reason why P_{cmax} and P_{bmax} take on opposite relationship against the parameter t . $t \rightarrow s$ leads to the completely failure of discrimination for Bob while $t \rightarrow 1$ means the extracting of all information for Bob and Charlie can't use the same qubit to discriminate the states.

The probability for both Bob and Charlie to identify the state can be obtained as

$$P^{SSD} = P_1(1 - q_1^b)(1 - q_1^c) + P_2(1 - q_2^b)(1 - q_2^c), \quad (6)$$

which satisfies the constraint $q_1^b q_2^b = s^2/t^2$ and $q_1^c q_2^c = t^2$. The optimal joint success probability P_S^{SSD} of Bob and Charlie has two candidates which can be achieved for different values of P_1 . The two cases are (i): $0 \leq P_1 < P_D$ and (ii): $P_D \leq P_1 \leq \frac{1}{2}$ with the optimal success probability

$$(i) : P_S^{SSD} = (1 - P_1)(1 - s)^2; \quad (7a)$$

$$(ii) : P_S^{SSD} = 1 - 4\sqrt{P_1 P_2}s + s; \quad (7b)$$

where $P_1 = P_D$ is the critical point corresponding to a continuous piecewise function and the maximum P_S^{SSD} is attained for $q_1^b = q_1^c = 1$, $q_2^b = q_2^c = s$, $t = \sqrt{s}$ and $q_1^b = q_1^c = \sqrt{\frac{P_2}{P_1}}s$, $q_2^b = q_2^c = \sqrt{\frac{P_1}{P_2}}s$, $t = \sqrt{s}$ corresponding to the two cases respectively. $t = \sqrt{s}$ as a necessary condition for optimal probability

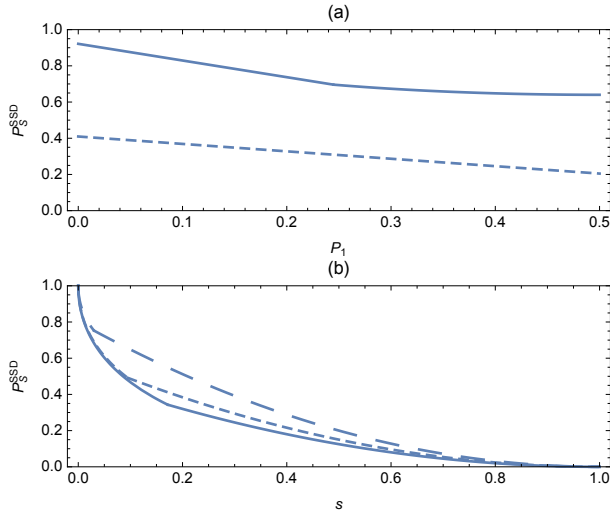


Fig. 2 The joint optimal success probability P_S^{SSD} as a function of the parameter P_1 (for $s = 0.04$ (solid line), 0.36 (dotted line) shown in Fig(a)) and s (for $P_1 = 0.5$ (solid line), 0.4 (dotted line), 0.2 (dashed line) as is shown in Fig(b)).

of SSD is the same as the equal prior case in [8]. The result is presented in Fig.2. We can find that the variation curve is symmetrical for $P_1 = 1/2$ (with the success probability minimized for equal prior case while maximized for extremely unequal prior case $P_1 = 1, 0$). This result shows that identifying unequal prior initial states is much easier than the equal prior case. P_S^{SSD} equals to 1, 0 for $s = 0, 1$ which means that we identifying two orthogonal initial states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ is extremely easy while distinguishing two identical states is impossible. For larger values of s , the possibility of existence of the result in the case (i) which means one of the initial states is ignored will be enlarged and the impact of *a priori* probabilities on the outcome is weakened which is shown in Fig.2(a). For different values of P_1 , the more unequal *a priori* probabilities of the initial states is, the tinier the possibility of existence of case (ii) will be and we are tend to be more probably to ignore one of the initial states which is shown in Fig.2(b). One can draw a conclusion that symmetry breaking in *a priori* probabilities of the prepared qubit leads to enhancement of the success probability of SSD on the contrary.

3 Comparison with other protocols

Then we begin to consider the success probability of another three protocols that allow Bob and Charlie to communicate classically with arbitrary *priori* probabilities and compare the results with SSD.

(1) Bob performs an optimal unambiguous discrimination measurement on the qubit he receives from Alice (which means all information of the qubit

extracted thus we should set $t = \langle \phi_1 | \phi_2 \rangle = 1$). No matter Bob succeeds or fails he tells Charlie the results; but if he fails, the procedure ends at that time. The optimal probability of both of them succeeding can be written as

$$(i) : P_S^{(1)} = (1 - P_1)(1 - s^2); \quad (8a)$$

$$(ii) : P_S^{(1)} = 1 - 2\sqrt{P_1 P_2 s}; \quad (8b)$$

for the two cases $0 \leq P_1 < \frac{s^2}{1+s^2}$ and $\frac{s^2}{1+s^2} \leq P_1 \leq \frac{1}{2}$ respectively.

(2) After the qubit is received from Alice, Bob performs an optimal unambiguous discrimination measurement on it. If he succeeds he will send a qubit which is in the same state as Alice has given him; but if he fails, the procedure ends. The probability of both of them succeeding can be written as $P^{(2)} = [P_1(1 - q_1^b) + P_2(1 - q_2^b)][P_1^c(1 - q_1^c) + P_2^c(1 - q_2^c)]$ where P_1^c and P_2^c are new *priori* probabilities of Charlie's qubit corresponding to the optimal discrimination of Bob with the constraints $q_1^c q_2^c = q_1^b q_2^b = s^2$. The optimal probability $P_S^{(2)}$ has two candidates which can be achieved for different values of P_1 . The two cases are (i): $0 \leq P_1 < \frac{s^2}{1+s^2}$ where $P_1^c = 0$, $P_2^c = 1$ and (ii): $\frac{s^2}{1+s^2} \leq P_1 \leq \frac{1}{2}$ which contains two situations: (a) $0 \leq P_1^c \leq \frac{s^2}{1+s^2}$ and (b) $\frac{s^2}{1+s^2} < P_1^c \leq \frac{1}{2}$ where $P_1^c = \frac{P_1 - \sqrt{P_1 P_2 s}}{1 - 2\sqrt{P_1 P_2 s}}$ and $P_2^c = \frac{P_2 - \sqrt{P_1 P_2 s}}{1 - 2\sqrt{P_1 P_2 s}}$. The corresponding optimal success probability $P_S^{(2)}$ can be written as

$$(i) : P_S^{(2)} = (1 - P_1)(1 - s^2)^2; \quad (9a)$$

$$(iia) : P_S^{(2)} = (P_2 - \sqrt{P_1 P_2 s})(1 - s^2); \quad (9b)$$

$$(iib) : P_S^{(2)} = (1 - 2\sqrt{P_1 P_2 s})(1 - 2\sqrt{P_1^c P_2^c s}). \quad (9c)$$

(3) Bob performs a probabilistic unitary optimal clone operation on the qubit he receives from Alice [21, 22] and Bob's cloning operation is shown as

$$U(|\Psi_i\rangle|0\rangle) = \sqrt{\gamma_i}|\Psi_i\rangle|\Psi_i\rangle|\alpha_i\rangle + \sqrt{1 - \gamma_i}|\beta\rangle|\beta\rangle|\alpha_0\rangle, i = 1, 2, \quad (10)$$

where $|0\rangle$ is a initialized state of the ancillas. $|\alpha_i\rangle$ and $|\alpha_0\rangle$ are orthogonal states of the flag corresponding to success or failure of the cloning respectively. γ_i is the success probability of the cloning for the state $|\Psi_i\rangle$ and $|\beta\rangle$ is a genetic failure state. The average success clone probability is $P_{clone} = P_1\gamma_1 + P_2\gamma_2$. The calculation details for optimal value of success cloning probability $P_{m(clone)}$ can be seen in Appendix.

If Bob succeeds in cloning, he keeps one of his clone qubits and sends the other one to Charlie and both of them perform optimal unambiguous discrimination to their qubits. If he fails he will inform Charlie and ends the

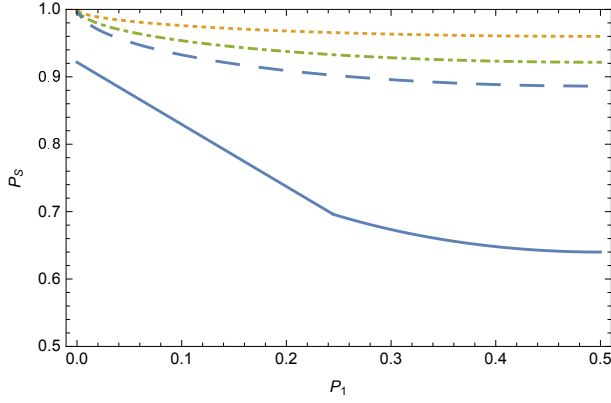


Fig. 3 The joint probability P_S as a function of P_1 is shown for $s = 0.04$ corresponding to the four strategies respectively. Solid line: P_S^{SSD} , dotted line: $P_S^{(1)}$, dot-dashed line: $P_S^{(2)}$, dashed line: $P_S^{(3)}$.

procedure. The probability of both of their succeeding is $P^{(3)} = P_{(clone)}P_bP_c$. Its optimal value can be derived

$$P_S^{(3)} = P_{m(clone)}P_{bmax}P_{cmax}, \quad (11)$$

where P_{bmax} and P_{cmax} equals to each other and they can be written as

$$(i) : P_{bmax} = (1 - P_1^0)(1 - s^2); \quad (12a)$$

$$(ii) : P_{bmax} = 1 - 2\sqrt{P_1^0 P_2^0 s}; \quad (12b)$$

with $P_1^0 = \frac{P_1\gamma_1}{P_1\gamma_1 + P_2\gamma_2}$, $P_2^0 = \frac{P_2\gamma_2}{P_1\gamma_1 + P_2\gamma_2}$ for the two cases $0 \leq P_1^0 < \frac{s^2}{1+s^2}$ and $\frac{s^2}{1+s^2} \leq P_1^0 \leq \frac{1}{2}$ respectively.

It's shown in Fig.3 that the minimum succeeding probability of discrimination corresponds to the equal prior states identification which is more difficult than the unequal case irrespective of which strategies we adopt. The protocol (1) has the most optimal success probability (the results in protocol (2),(3) differs only a little from protocol (1)) while the SSD strategy attains the weakest. Compared with the results for $P_1 = P_2 = 1/2$ [20], the success probability difference between SSD and the other three strategies is reduced for unequal prior cases.

Then we consider the probability that at least one of the parties succeeds in identifying the states (shown in Fig.4). For SSD protocol, we have known that $P^{SSD*} = P_1(1 - q_1^b q_1^c) + P_2(1 - q_2^b q_2^c)$ with the same constraints as Eq.(6). The optimal probability is attained $P_S^{SSD*} = P_S^{(1)}$. The results is the same for protocol (1), (2). But for the cloning protocol, $P^{(3)*} = P_c[1 - (P_1^0 q_1^b +$

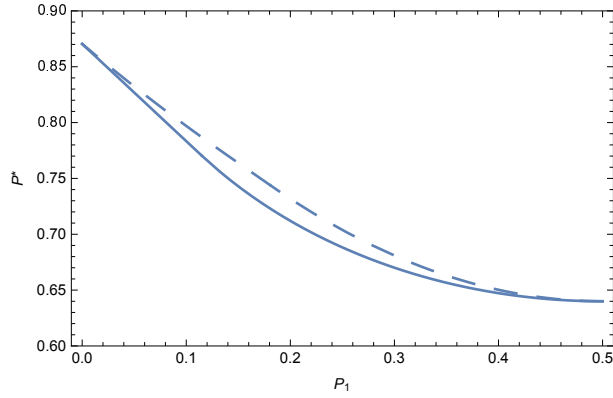


Fig. 4 The probability P^* for one of the two parties succeeds as a function of P_1 for four strategies. Solid line: P_S^{SSD*} , $P_S^{(1)*}$, $P_S^{(2)*}$, dashed line: $P_S^{(3)*}$

$P_2^0 q_2^b)(P_1^0 q_1^c + P_2^0 q_2^c)]$ with the constraint $q_1^b q_2^b = q_1^c q_2^c = s^2$. Its optimal value $P_S^{(3)*}$ has two candidates which can be achieved for different values of P_1^0 .

The two cases are (i): $0 \leq P_1^0 < \frac{s^2}{1+s^2}$ and (ii): $\frac{s^2}{1+s^2} \leq P_1^0 \leq \frac{1}{2}$ with the maximal success probability

$$(i) : P_S^{(3)*} = P_{max}[1 - (P_1^0 + P_2^0 s^2)^2]; \quad (13a)$$

$$(ii) : P_S^{(3)*} = P_{max}(1 - 4P_1^0 P_2^0 s^2). \quad (13b)$$

The outcomes in [20] corresponding to the equal prior cases indicates that $P_S^{(1)*} = P_S^{(2)*} = P_S^{(3)*} = P_S^{SSD*} = 1 - s$ which is a special case for our results (shown in Fig.4). The clone protocol presents some slight advantages in the success probability of discrimination compared with the other three which is equivalent to the results of $P_S^{(1)}$. When P_1 approaches 0, 1, 1/2, the results of the four protocols equal to each other.

Then which protocol will do better? If we require both Bob and Charlie succeeding, the protocol (1) will do best. But for unequal prior case, the difference between the protocol (1) and the other three shrinks a lot. While if we require at least one of them succeeds, the situation is almost the same for the four strategies. However, we should mention a fact that SSD protocol uses only one qubit while the other three strategies use two. For the overall consideration of economical use of quantum information source and high efficiency of state discrimination, we take SSD protocol as our better choice.

4 Correlations

The results in [8] indicates that the success probability of SSD is positively correlated with quantum correlations for the special equal prior case. Then for the general unequal prior one, what is the relation of the success probability of SSD, arbitrary *priori* probabilities P_1 , P_2 and the quantum correlation between A and B ? That's the main question we now focus on. Choosing $|\phi_i\rangle = |\chi_i\rangle$, after Bob's unitary operation, the state of our system which contains the ancilla can be written as

$$\begin{aligned}\rho_{AB} &= U_b[P_1|\Psi_1\rangle\langle\Psi_1| \otimes |0\rangle_b\langle 0|_b + P_2|\Psi_2\rangle\langle\Psi_2| \otimes |0\rangle_b\langle 0|_b]U_b^\dagger \\ &= P_1|\phi_1\rangle\langle\phi_1| \otimes |\alpha_1\rangle_b\langle\alpha_1|_b + P_2|\phi_2\rangle\langle\phi_2| \otimes |\alpha_2\rangle_b\langle\alpha_2|_b.\end{aligned}\quad (14)$$

This is a mixed separable state obviously. A and B are completely disentangled. Apart from this, we turn to a kind of correlation-quantum discord which contains the right discord D_{AB} and the left discord D_{BA} [8].

For the two-rank system ρ_{AB} , by using the Koashi-Winter identity [23], its discord can be can be obtained as a reduced state of the tripartite state

$$|\Psi\rangle = \sqrt{P_1}|\phi_1\rangle|\alpha_1\rangle_b|0\rangle_d + \sqrt{P_2}|\phi_2\rangle|\alpha_2\rangle_b|1\rangle_d, \quad (15)$$

where $|0\rangle_d, |1\rangle_d$ is the basis of a fictitious qubit D . The right discord can be expressed as

$$D(\rho_{AB}) = S(\rho_B) - S(\rho_D) + E(\rho_{AD}), \quad (16)$$

where $S(\rho_B)$, $S(\rho_D)$ and $E(\rho_{AD})$ correspond to the reduced entropy of B , D and the entanglement of formation between A and D respectively. Go to a step further, it can be expressed explicitly as

$$D(\rho_{AB}) = F(\tau_B) - F(\tau_D) + F(\tau_A - \tau_{ABD}), \quad (17)$$

where

$$F(x) = -\frac{1 + \sqrt{1-x}}{2} \log_2 \frac{1 + \sqrt{1-x}}{2} - \frac{1 - \sqrt{1-x}}{2} \log_2 \frac{1 - \sqrt{1-x}}{2}. \quad (18)$$

τ_{ABD} is the residual tangle of the tripartite state [24], τ_A (similarly for τ_B and τ_D) is the tangle between A and B , D . They can be obtained by

$$\begin{aligned}\tau_{ABD} &= 4P_1P_2(1-t^2)(1-r^2), \tau_A = 4P_1P_2(1-t^2), \\ \tau_B &= 4P_1P_2(1-r^2), \tau_D = 4P_1P_2(1-t^2r^2),\end{aligned}\quad (19)$$

where we set $r = s/t = \langle\alpha_1|\alpha_2\rangle$. We can also get the left discord $D(\rho_{BA})$ easily by interchanging the subscript A and B in Eq.(17). To analyze their roles in SSD, we define the proportion of left (right) discord in their total

$$\tilde{D}_{left} = \frac{D(\rho_{BA})}{D(\rho_{AB}) + D(\rho_{BA})}, \tilde{D}_{right} = \frac{D(\rho_{AB})}{D(\rho_{AB}) + D(\rho_{BA})}, \quad (20)$$

and a symmetrized discord

$$D_{\text{symm}} = \sqrt{D(\rho_{BA})D(\rho_{AB})}. \quad (21)$$

It is obviously shown in Fig.5(a) that the proportion of left (right) discord in their total is positively (negatively) correlated with the parameter t which corresponds to the information extracted by Bob after his measurement. Both \tilde{D}_{left} and \tilde{D}_{right} change very little toward the *priori* probability P_1 (shown in Fig.5(b)). The maximum symmetrized discord of initial state is obtained at $P_1 = P_2 = 1/2$ which is shown in Fig.5(c) while the success probability of SSD becomes weakest on the contrary. The deviation from the equal prior case for the initial state enhances the success probability of quantum state discrimination but destroys both the right and the left discord between A and B conversely. When P_1 approaches 0 or 1, the system-ancilla state turns into a pure product state with two parts have no quantum correlation with each other and both left and right discord equals to 0 (optimal success probability of SSD can be derived there according to the results in Fig.2). Hence, we can draw a conclusion that distinguishing the initial states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ prepared in equal priors is most difficult and requires more quantum correlation than the unequal prior case.

Other calculations show that both left and right discord also equal to 0 for $s \rightarrow 0, 1$. Because identifying two orthogonal initial states is much easier while distinguishing two identical states is impossible, these two procedures don't require any quantum correlation.

For a fixed value of s , we set $t = s^{1/n}$. Through a simple calculation, we have known that $n = 1$ ($t = s$) corresponding to a complete failure of discrimination for Bob also makes A and B uncorrelated. When $n = 2$ ($t = \sqrt{s}$), left discord equals to the right one. For arbitrary *priori* probabilities, $t = \sqrt{s}$ as a necessary condition for optimal value of both quantum correlation and success probability of SSD is the same as the results for special equal prior case. Thus, the conclusions in [8] are generalized here. The further increase of n (t increases as well) signifying more information extracted induces a much more severe destroying effect on the right discord than the left one. When $n \rightarrow \infty$ ($t \rightarrow 1$), we can easily obtain $D_{BA} = D_{AB} = 0$ which signifies the complete death of the quantum correlation between A and B . This result means that Bob performs an optimal state discrimination and extracts all information while Charlie could do nothing except resorting to classical communication as is mentioned in the above three protocols if he wants to know the result.

So one can draw a conclusion that quantum correlation disappears in the following three cases: (1) initial states prepared with extremely unequal prior ($P_1 \rightarrow 0, 1$); (2) orthogonal initial states ($s = 0$) to be distinguished; (3) the optimal state discrimination performed by Bob ($t = 1$). The deviation for the overlap $\langle \psi_1 | \psi_2 \rangle$, $\langle \alpha_1 | \alpha_2 \rangle$ and the *priori* probability P_1 from their maximum and minimum may avoid the occurrence of quantum correlation death between A and B .

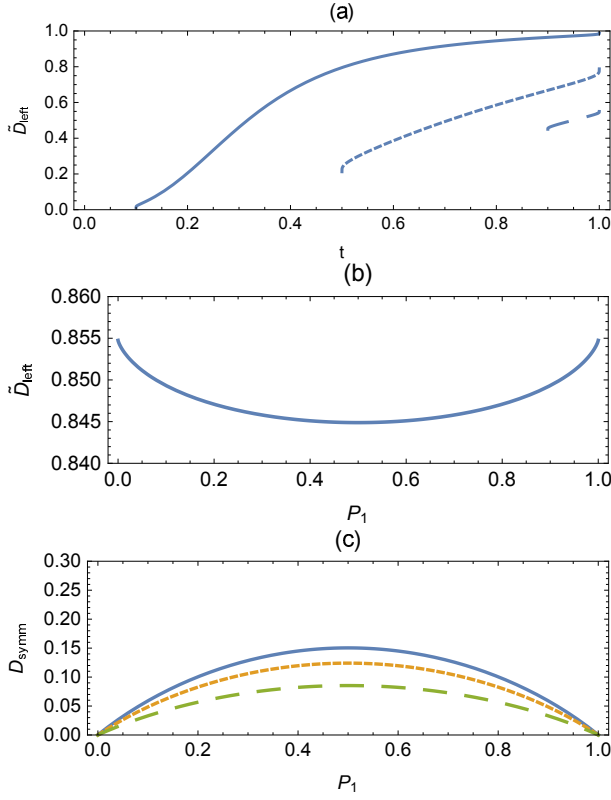


Fig. 5 \tilde{D}_{left} as a function of the parameter t for $P_1 = 0.2$, $s = 0.1$ (solid line), 0.5 (dotted line), 0.9 (dashed line) respectively (shown in Fig(a)) and P_1 for $s = 0.1$, $t = s^{1/4}$ (shown in Fig(b)); Fig(c) corresponds to the symmetrized discord as a function of the *priori* probability P_1 for $s = 0.36$, $t = s^{1/2}$ (solid line), $t = s^{1/4}$ (dotted line) and $t = s^{1/8}$ (dashed line) respectively.

5 Summary

We study the procedure of SSD with ancillas, in which the two nonorthogonal states prepared for arbitrary *priori* probabilities. The success probability of SSD is found to be enhanced by the *priori* knowledge of observers about the chances of the two states. Then we compare the results with the protocols that allow classical communication, it indicates that SSD protocol does better than the one allowing classical communication. The following two conclusions about the quantum correlations in system-ancilla state can be drawn. Both left and right discord are destroyed in unequal prior cases and reach their maximums when $P_1 = P_2 = 1/2$, which demonstrates opposite tendency for the success probability of SSD against P_1 ; The proportion of left (right) discord in their total is positively (negatively) correlated with the information extracted by Bob, which is found in the case with equal *a priori* probabilities [8].

The death of quantum correlation between A and B occurs for the following three cases:

- (a) extremely unequal prior cases with $P_1 \rightarrow 0$ or 1;
- (b) optimal ambiguous discrimination measurement by Bob where $t \rightarrow 1$;
- (c) orthogonal initial states ($s = 0$).

The cases (a) and (c) enhance the success probability of SSD, while the case (b) makes the joint identification of the states impossible except resorting to classical communication. At last, we can draw a conclusion that identifying unequal prior initial states calls for less quantum correlation than the equal prior case.

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A Calculations of probabilistic optimal cloning

According to Eq.(10), it's easily found that

$$s = \sqrt{\gamma_1 \gamma_2} s^2 \langle \alpha_1 | \alpha_2 \rangle + \sqrt{(1 - \gamma_1)(1 - \gamma_2)}. \quad (22)$$

where $|\alpha_1\rangle = |\alpha_2\rangle$ is required for optimal cloning[21].

If we set $\sin \theta_i = \sqrt{1 - \gamma_i}$ ($i = 1, 2$) for $0 \leq \theta_i \leq \pi/2$ and the variables $x = \cos(\theta_1 + \theta_2)$, $y = \cos(\theta_1 - \theta_2)$ are further introduced. Eq.(22) is equivalent to $2s = (1 + s^2)y - (1 - s^2)x$. And then we find an intermediate parameter ω which satisfies

$$x = \frac{1 - (1 + s^2)\omega}{s}, y = \frac{1 - (1 - s^2)\omega}{s}. \quad (23)$$

The range of the parameter ω is given in Eq.(27). Then it's found that

$$\gamma_i = \frac{1}{2} [1 + xy + (-1)^i \sqrt{(1 - x^2)(1 - y^2)}]. \quad (24)$$

To find the maximum value $P_{m(clone)}$, the following equation should be satisfied $P'_{clone} = \frac{dP_{clone}}{dt} = 0$ which is equivalent to $P_1 \gamma'_1 + (1 - P_1) \gamma'_2 = 0$, thus the final results can be obtained

$$P_1 = \frac{\gamma'_2}{\gamma'_2 - \gamma'_1}, P_{m(clone)} = \frac{\gamma'_2 \gamma_1 - \gamma'_1 \gamma_2}{\gamma'_2 - \gamma'_1}. \quad (25)$$

where

$$\gamma'_i = \frac{d\gamma_i}{dt} = \frac{\sqrt{\gamma_i(1 - \gamma_i)}}{s} \left[-\frac{1 + s^2}{\sqrt{1 - x^2}} + (-1)^i \frac{1 - s^2}{\sqrt{1 - y^2}} \right]. \quad (26)$$

It indicates that both the *priori* probability P_1 , P_2 and the optimal cloning probability $P_{m(clone)}$ can be considered as a function of the parameter ω with the range

$$\omega_1 \leq \omega \leq \omega_2, \omega_1 = \frac{1}{1 + s}, \omega_2 = \frac{1}{1 + s^2}. \quad (27)$$

where ω_1 and ω_2 correspond to the cases for $P_1 = P_2 = \frac{1}{2}$ and $P_1 = 0$ respectively. Hence, the optimal value of both P_{bmax} and P_{cmax} in Eq.(11),(12) can be obtained as parametric equations of ω .

